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Brief communication

The falling velocity of a large sphere in a suspension of smaller spheres

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1. Introduction

The determination of the solid–fluid interaction force in a suspension is still the subject of theoretical and experimental investigation, both in academia and in industry. To this end, large bodies of experimental evidence have been produced, especially regarding mono-component solid systems and, as a consequence, many empirical correlations have been proposed (e.g., Di Felice, 1994). Systems made up of two or more solid types, although being of practical relevance, have been investigated less copiously, given the intrinsic difficulties in carrying out the experiments and in interpreting the results.

Rigorous theoretical approaches, for mono-component as well as multi-component solid mixtures, are possible only for systems under viscous flow regime conditions and are effectively limited to dilute suspensions. Notably, Batchelor (1982) has derived a linear relation for the settling velocity of a sphere as a function of the concentration ϕ of the other spheres in a suspension:

$$u_i = u_{i,0}\left(1 + \sum_j S_{i,j}\phi_j\right), \tag{1}$$

where $u_{i,0}$ is the isolated sphere settling velocity. In a subsequent work, Batchelor and Wen (1982) estimated numerical values of the coefficient $S_{i,j}$ for specific values of the dimensionless parameters λ and γ:

$$\lambda = \frac{d_j}{d_i} \quad \text{and} \quad \gamma = \frac{\rho_j - \rho}{\rho_i - \rho} \tag{2}$$

with d and ρ representing particle diameter and density, respectively.

A satisfactory, if limited, verification of Batchelor's theoretical predictions for non-colloidal particles has been previously carried out by Davis and Birdsell (1988) in a study of settling velocities of 261 μm and 136 μm glass ballotini (i.e., λ of about 2 and γ equal to 1) and by Di Felice and Pagliai (2003) for single spheres settling in a variety of suspensions of different spheres. Here, we extend these results by concentrating attention on specific systems characterized by very small values of λ, for which Batchelor and Wen (1982) were able to suggest a general simple relationship quantifying the coefficient $S_{i,j}$, namely:

$$S_{i,j} = -2.5 - \gamma. \tag{3}$$

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2. Experimental

The experiments were carried out by simply dropping a single sphere into a viscous suspension of other particles and measuring the steady-state settling velocity over a fixed length by means of a stop-watch. The fluid was silicon oil of 970 kg/m^3 density and 12 Pa s viscosity. Eight different systems were investigated, all characterized by a very small λ values, as reported in Table 1.

The experimental procedure was very similar to that reported in the previous work (Di Felice and Pagliai, 2003) and therefore will not be repeated here; the only major change being due the fact that some of the suspensions were not transparent, so that settling particles could not be tracked by the naked eye. In these cases, the suspension was first prepared in a separate vessel, then gently poured into the sedimenting column over a layer, some 50 mm high, of a clear dense liquid (methylene iodide $\rho = 3350$ kg/m^3), thereby enabling particles placed on the top surface of the bed to be observed on arrival at the bottom, so permitting their journey time to be easily measured (see Fig. 1). Measured settling velocities have been analysed relative to the measured settling velocity in pure fluid, thereby obviating the necessity of considering the possible effect of the container wall, this being assumed to be the same both in absence and presence of the suspended particles, as suggested by Brenner et al. (1990).

3. Results

Fig. 2 reports experimental settling velocities as measured for a typical system (system #8: lead in a tungsten particle suspension). The Figure confirms the known results of linear dependency of settling velocity on particle suspension concentration. A linear fit of the data yields the experimental value of the coefficient $S_{i,j}$, which

Table 1
Physical properties of the solid used

#	Test sphere			Suspension sphere			λ	γ
	Material	Density (kg/m^3)	Size (mm)	Material	Density (kg/m^3)	Size (mm)		
1	Plastic	1280	4.85	Glass	2550	0.053	0.010	5.09
2	Glass	2550	6	Glass	2550	0.053	0.008	1
3	Lead	11,400	3.35	Glass	2550	0.053	0.015	0.15
4	Lead	11,400	3.35	Lead	11,400	<0.04	<0.015	1
5	Zirconia	3800	2.48	Lead	11,400	<0.04	<0.015	3.68
6	Lead	11,400	3.35	Copper	8800	<0.04	<0.015	0.75
7	Glass	2550	6	Zirconia	3800	0.7	0.11	1.79
8	Lead	11,400	3.35	Tungsten	19,300	<0.010	<0.003	1.75

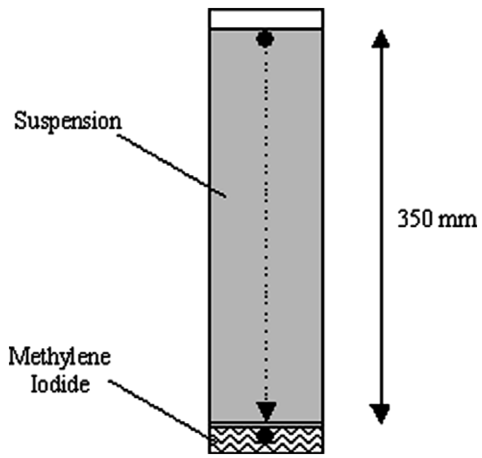

Fig. 1. Sketch of the experimental apparatus with a layer of heavy liquid underneath the suspension.

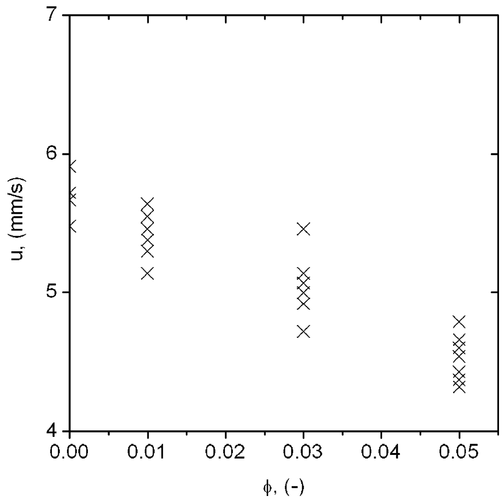

Fig. 2. Experimental settling velocity for system #8.

for all the systems investigated is reported and compared with Batchelor's predictions in Fig. 3: agreement is excellent, thus further increasing confidence in the theoretical predictions.

The same results can also be interpreted in terms of the less rigorous but more intuitive approach of a *pseudo-fluid*. Assuming the large sphere to be settling through a pseudo homogenous fluid (made up of the fluid itself and the smaller suspended particles), of quantifiable apparent density and apparent viscosity, then we can write, for viscous flow conditions:

$$u_i = u_{i,0}\frac{\mu}{\mu_{\rm ps}}\frac{(\rho_i - \rho_{\rm ps})}{(\rho_{-}\rho)}. \tag{4}$$

The pseudo-fluid apparent density is simply

$$\rho_{\rm ps} = \phi\rho_j + (1-\phi)\rho \tag{5}$$

and pseudo-fluid apparent viscosity, with the assumption of a linear dependency on suspension volume fraction, may be written

$$\mu_{\rm ps} = \mu(1+k\phi). \tag{6}$$

On this basis we obtain:

$$u_i = u_{i,0}\frac{(1-\gamma\phi)}{(1+k\phi)}, \tag{7}$$

which for small values of ϕ is equivalent to

$$u_i = u_{i,0}[1+(-k-\gamma)\phi]. \tag{8}$$

Comparing this result with that of Batchelor (Eqs. (1) and (3)) results in a value for k of 2.5, thereby converging on the rigorous theoretical result of Einstein (1906, 1911) for vanishingly dilute, neutrally buoyant suspensions. For neutrally buoyant suspensions, the measured apparent viscosity is independent of the diameter of the falling sphere – so long as this is greater than the diameter of the suspended particles, $\lambda < 1$ (Milliken et al., 1989); for non-neutrally buoyant suspensions, however, this convenient assumption may no longer be valid. Needless to say, the above numerical finding for the suspension viscosity is only valid for very dilute conditions; in the case of higher particle concentrations other relationships must be used, such as the empirical relation of Thomas (1965):

$$\mu_{\rm ps} = \mu(1+2.5\phi+10.05\phi^2+0.00273\exp(16.6\phi)) \tag{9}$$

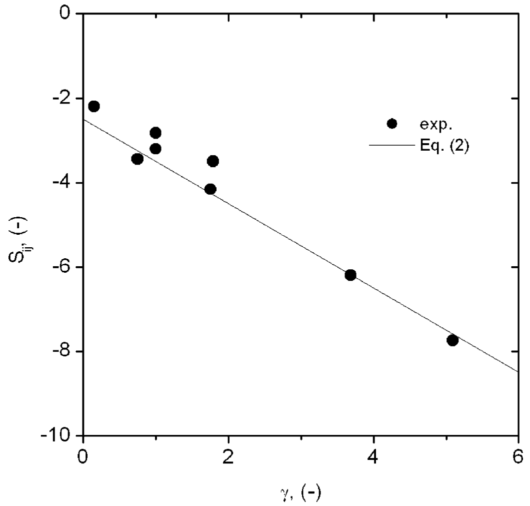

Fig. 3. Experimental value of the coefficient S_{ij} compared with Batchelor's theoretical predictions.

or the semi-theoretical derived expression of Gibilaro et al. (2007):

$$\mu_{ps} = \mu(1-\phi)^{-2.8}. \tag{10}$$

Both Eqs. (9) and (10) are in good agreement with the present results for dilute conditions.

Finally, it should be remarked that although viscosity is an intrinsic material property of Newtonian fluids, the same cannot be said for the "apparent" viscosity of suspensions, the experimentally determined values of which appear to be dependent on the method used. (Reardon et al., 2005).

4. Conclusions

This work has shown further support to Batchelor's theoretical predictions for the settling velocity of sphere in multicomponent solid suspensions in viscous flow regime. Moreover, the same results have confirmed the applicability of the pseudo-fluid simplification with the pseudo-fluid viscosity given by Einstein relationship even for non-neutrally buoyant suspensions.

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